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EXPERIMENTAL IDENTIFICATION OF NON-STATIONARY SELF-HEATING CHARACTERISTICS OF LAMINATED COMPOSITE PLATES UNDER RESONANT VIBRATION

This paper presents experimental studies on the thermal response of laminated composite plates under resonant vibrations. The thermoviscoelastic behaviour of laminated glass-fibre reinforced polymer (GFRP) composite specimens was studied. The specimens were subjected to purely flexural bending cyclic loading conditions on their three first bending resonant frequencies. During the examination of the specimens, frequency response functions (FRF) and thermal responses were evaluated. Infrared images acquired during the experiments also allowed the study of the temperature profiles of the specimens and temperature evolution over the loading time. The maximal magnitudes of temperature were observed at a point located on specimens' clamp line, which was caused by the maximal stress magnitudes. The temperature evolution curves revealed the double-exponential characteristic which was affected by the evolution of the dynamic moduli of the material during resonant vibration. The temperature increased until the equilibrium between the dissipated energy and energy convected to the environment was reached. Based on the measurement data, the empirical model of self-heating temperature evolution was proposed. The influence of the excitation frequency and the plate length on the obtained temperature distributions was also examined. It was noticed that the excitation frequency was linearly dependent and the plate length was power dependent on the self-heating temperature, which confirms the numerical results obtained in previous theoretical studies. The obtained results could be used for the prediction and prevention of composite structural degradation during resonant cyclic constant-strain loading.

Keywords: **polymeric layered composites, resonant vibrations, self-heating temperature, infrared thermography**

EKSPERYMENTALNA IDENTYFIKACJA CHARAKTERYSTYK NIEUSTALONEGO SAMOROZGRZANIA LAMINATOWYCH PŁYT KOMPOZYTOWYCH PODCZAS DRGAŃ REZONANSOWYCH

W artykule przedstawiono wyniki badań eksperymentalnych dotyczące zachowania termolepkosprężystego laminatów polimerowych zbrojonych włóknem szklanym poddanych cyklicznym obciążeniom gnącym o częstotliwościach pokrywających się z pierwszymi trzema częstotliwościami własnymi wyznaczonymi na podstawie badań modalnych. Podczas eksperymentów badano również odpowiedź cieplną, co pozwoliło na wyznaczenie profili temperaturowych oraz ich zmian w czasie. Zgodnie z wcześniej przeprowadzonymi badaniami teoretycznymi, maksymalne wartości temperatury zaobserwowano w linii utwierdzenia próbek, co jest spowodowane maksymalnymi wartościami naprężeń w tym obszarze. Charakter zmian temperatury w czasie można opisać za pomocą sumy dwóch funkcji ekspotencjalnych, co wynika ze zmian modułów dynamicznych materiału. Temperatura wzrastała do momentu bilansu pomiędzy energią dyssypowaną a energią odprowadzaną do otoczenia. Na podstawie danych pomiarowych zaproponowano model zmiany temperatury samorozgrzania. W trakcie badań zbadano również wpływ częstotliwości wymuszenia oraz długości płyty na otrzymane rozkłady temperatury. Zaobserwowano, że częstotliwość wymuszenia jest w liniowej zależności, a długość płyty - w potęgowej zależności z temperaturą samorozgrzania, co potwierdza wyniki numeryczne otrzymane podczas wcześniejszych badań teoretycznych. Uzyskane wyniki mogą być wykorzystane do predykcji i zapobiegania degradacji strukturalnej kompozytów podczas rezonansowych cyklicznych obciążeń ze stałymi odkształceniami.

Słowa kluczowe: **polimerowe kompozyty warstwowe, drgania rezonansowe, temperatura samorozgrzania, termowizja**

INTRODUCTION

Technological progress in material engineering, especially in the domain of fibre and textile reinforced plastics, makes possible the replacement of parts made of metal alloys with parts of reinforced composites in the process of machinery design. High specific strength and stiffness along with the possibility of controlling their properties through reinforcement and matrix optimization are important advantages of these composites.

Many composite structures are subjected to vibrations which could be extremely dangerous when their frequency covers resonant frequency. During resonant vibrations, due to cyclic loading, regions of high stress occur. The location of these regions corresponds to the location of the antinodes of the eigenmodes. According to the thermoviscoelastic behaviour of polymeric composite structures [1] the energy density created in high stress regions is transformed into a local change of temperature due to the dissipation process, which could be measured on the material surface. The temperature change is proportional to the change of stress. It corresponds to the irreversible exchange between mechanical energy and heat. Due to the poor thermal diffusivity of polymers, the temperature rises until the structure reaches a thermal steady-state, which is exhibited as a balance between the heat generation and heat convection to the environment. If we additionally take into consideration the dynamic moduli evolution, the self-heating causes irreversible local structural degradation, whose propagation could affect a catastrophic breakdown of the structure [2]. Therefore, it is of considerable importance to investigate the influence of the self-heating effect on the degradation of polymeric composites.

The research in this field has been carried out by various scientific institutions. The theoretical aspects of the phenomenon of thermoviscoelasticity were developed by Karnaukhov and Senchenkov [3]. The authors of [4-6] show the theoretical and experimental results of the self-heating effect during fatigue tensile tests of polymeric composites. The self-heating of cyclically loaded polymeric composites with defects was discussed in [7]. Song et al. [8] present the results of the self-heating of an epoxy syntactic foam during compressive loading.

In the article, experimental research devoted to the identification of a model of the self-heating effect was carried out. During the research, GFRP rectangular plates of various lengths were subjected to excitation by harmonic vibration with frequencies corresponding to the plate's natural frequencies. In order to identify the natural frequencies of the specimens, a preliminary modal analysis was carried out. During the vibration test the temperature on the specimen surfaces was measured with the use of an infrared camera. Recorded sequences of the infrared images allowed identification of the thermal characteristics at selected points of the plates. The problem of resonant self-heating of composites was discussed in [9]. However, the authors of this article present the phenomenon from a new point of view. Furthermore, the approximation model of non-stationary self-heating will be shown. Moreover, the connections and dependencies of stiffness complex parameters evolution and heating-up of the structure were investigated. The obtained results will be helpful in the research on fatigue and fracture of polymeric composites.

THEORETICAL BACKGROUND

The linear viscoelastic behaviour of polymeric composites taking into consideration the self-heating effect could be presented in terms of the Boltzmann superposition principle. Considering the multiple stress relaxations of the investigated laminated composite [10] the stress relation could be given with use of the Maxwell-Weichert rheological model as follows:

$$\sigma(t) = \varepsilon(t)E_0(\theta) - \sum_{i=1}^{n} \int_0^t \varepsilon(\tau_i) R_i(t-\tau_i,\theta)\mathrm{d}\tau_i \tag{1}$$

where i is the number of Maxwell elements connected in parallel, $\sigma(t)$ and $\varepsilon(t)$ are the time-dependent stress and strain respectively, θ is the temperature, $E_0(\theta)$ is the temperature-dependent equilibrium Young's modulus, τ_i are the relaxation times and $R_i(t-\tau_i,\theta)$ are the temperature-dependent relaxation kernels. Here and further we assume the cycle-averaged temperature θ_a due to the small temperature variations over the cycle [11]:

$$\theta_a(X) = \frac{\omega}{2\pi} \int_t^{t+T} \theta(X)\mathrm{d}t \tag{2}$$

where X is the vector of Cartesian coordinates of the plate, ω is the angular velocity and T is the cycle period. The stress-strain relationship for the investigated material in a frequency domain, taking into consideration the temperature dependence, could be presented using the approach of complex parameters (cf. [12]):

$$\tilde{\sigma}(\omega) = E^*(\omega,\theta)\tilde{\varepsilon}(\omega) \tag{3}$$

where $(\tilde{\cdot})(\omega)$ denotes the Fourier transform for $(\tilde{\cdot})(t)$ and E^* is the complex modulus which could be decomposed as:

$$E^*(\omega,\theta) = E'(\omega,\theta) + iE''(\omega,\theta) \tag{4}$$

where E', E'' are storage and loss moduli respectively, which physically denote elastic and viscous response with respect to the generalized Maxwell model of viscoelasticity. The most typical way of evaluating complex moduli is by performing dynamic mechanical analysis. In [13] the authors present the master curve for the loss modulus for the investigated composite material which allows one to obtain the value of E'' for various temperatures and frequencies.

In order to obtain the temperature distribution on the surface of the investigated plates, it is necessary to solve the heat transfer problem. Assuming non-stationarity, the problem is presented by the heat transfer equation with appropriate convective boundary conditions:

$$c(\theta)\rho(\theta)\frac{\partial\theta(X,t)}{\partial t} - \nabla\cdot\left[\lambda(\theta)\nabla\theta(X,t)\right] = Q_{sh}(X,t) \tag{5}$$

where: c is the temperature-dependent specific heat, ρ is the temperature-dependent material density, λ is the temperature-dependent thermal conductivity and $Q_{sh}(X,t)$ is the source function, ∇ is a vector differential operator with respect to the coordinate vector X. The source function in (6) is the energy dissipated due to the hysteretic behaviour of the viscoelastic material:

$$Q_{sh}(X,t)=\frac{\omega}{2\pi}\int_{0}^{2\pi/\omega}\sigma\dot{\varepsilon}\,\mathrm{d}t \qquad (6)$$

Equation (5) could not be solved analytically, therefore, experimental investigations were performed to determine the temperature distribution on the surface of the plates subjected to various loading conditions. The analytical solution of the steady-state self-heating for the presented problem could be found in [11]. The experimental investigation of the temperature evolution during self-heating allowed us to explain the viscoelastic structural behaviour during oscillatory loading with constant strain.

SPECIMENS AND EXPERIMENTAL SETUP

A series of experiments were carried out in order to identify the temperature distributions of rectangular plates made of GFRP. The experiments were performed on a laboratory stand (Fig. 1) and consisted of the following components:

1. Specimen holder *7* which allowed for mounting of the specimen under repeatable conditions. In addition, the holder provided insulation from heat produced during self-heating effect.
2. Specimen *4* which was manufactured from epoxy E-glass prepreg PR-UD EST 250/635 FT102 35 supplied by *Epo* GmbH with the following lay-up $[0/\pm 60/\mp 60/0]_S$ and the following properties [11]: E_1 = 38.283 GPa, E_2 = 10.141 GPa, G_{12} = = 3.533 GPa, ν_{12} = 0.366, ρ = 1794 kg/m^3. Specimen width (W) and thickness (H) were respectively W = 10±0.1 mm, H = 2.5±0.05 mm. Different cases of specimen length L were considered: 200, 225, 250, 275, 300 mm. Total number of investigated plates was 30. In a further notation we will use effective length L_e, which presents length of specimen without clamped region: $L_e = L - 20$ mm
3. Electrodynamic shaker *1* with a power amplifier *10* which was driven by generator *9*. Shaker was connected with specimen *4* through stinger *2* with small specimen clamp *3* at its end
4. Single point laser Doppler vibrometer (LDV) *6* connected to multi-channel signal analyzer *8* was used for rapid and non-contact measurement and for controlling excitation parameters
5. IR-camera *5* applied to temperature measurements on specimen surface during its resonant vibration. During each test, sequence of IR images was recorded for further investigation of self-heating effect. Images were acquired with frame rate of 1 f/s with use of specialized software installed on PC *11*

During the experimental test, the specimen was excited to harmonic vibration with a frequency equal to the earlier identified [10] resonant frequency.

The tests were performed for the first three resonant frequencies. For each case of specimen length and each considered resonant frequency, two specimens were tested.

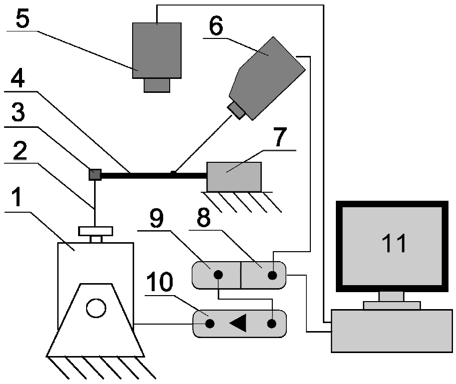

Fig. 1. Experimental setup

Rys. 1. Stanowisko badawcze

RESULTS AND DISCUSSION

The recorded sequences of infrared images were analyzed in order to identify the temperature characteristics connected to the self-heating effect. In Figure 2 the exemplary thermogram recorded for a plate vibrating at the first natural frequency was shown. It is clearly visible that the hot spots are placed close to the clamps and in the antinode areas. A temperature profile (Fig. 3) identified along line (L1) covering the center line of the specimen confirms the location of higher temperature areas connected with the self-heating effect.

Temperature characteristics were evaluated in points which were defined on the infrared image (Fig. 2). Figure 4 (dotted line) shows the exemplary curve of the peak temperature values identified at point (P4) located close to the clamp line.

In order to observe the heating effect, a 3D plot (Fig. 5.) showing the time evolution of the temperature values measured along profile line (L4) parallel to the clamping line were additionally generated.

One can observe that the intensity of heating depends on the location of the measurement point. The temperature reached the highest values in the region situated nearest the plate support, which corresponded to maximal stress magnitudes in this region. The lowest values occur in the antinode area in the middle of the plate. Similar results were obtained for all the investigated specimens. As shown in Figure 5, the temperature along the profile is not regularly distributed. It is caused by non-uniform clamping of the specimen. The shape of the temperature distribution along the profile strongly depends on the clamping conditions. The peak temperature evolutions determined for point P4 located on the plate clamp line are considered below.

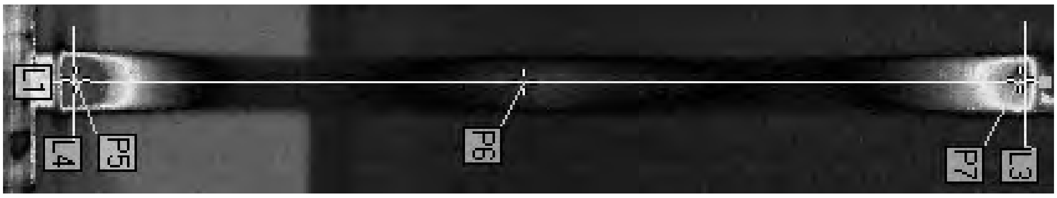

Fig. 2. Infrared image of plate excited at resonant frequency

Rys. 2. Termogram próbki podczas drgań rezonansowych

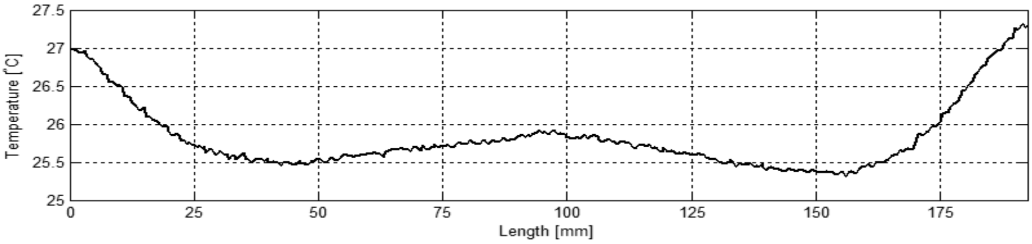

Fig. 3. Exemplary vertical temperature profile of investigated plate corresponding to first natural frequency

Rys. 3. Przykładowy pionowy profil temperaturowy badanego obiektu odpowiadający pierwszej częstotliwości własnej

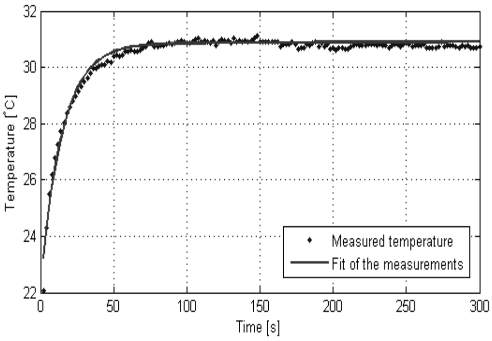

Fig. 4. Exemplary plot of temperature evolution for specimen of L_e = 180 mm during vibration on first resonant frequency f_1 = = 262 Hz and approximation

Rys. 4. Przykładowy wykres przyrostu temperatury dla próbki z L_e = = 180 mm podczas drgań przy pierwszej częstotliwości rezonansowej f_1 = 262 Hz i aproksymacja

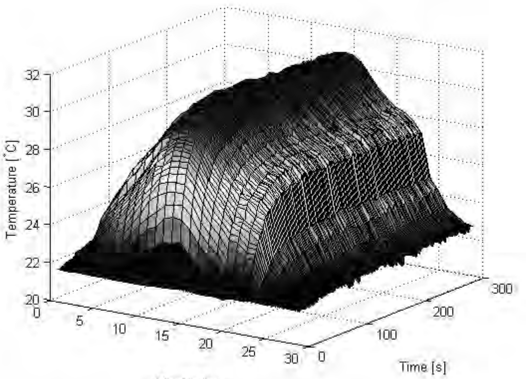

Fig. 5. Plot of temperature profiles during vibration at first resonant frequency

Rys. 5. Wykres profili temperaturowych podczas drgań przy pierwszej częstotliwości rezonansowej

Considering the character of the obtained temperature curves it is evident that they behave as exponential functions. For the purposes of approximation, a curve fitting the use of a double exponential function (7) was performed

$$\theta(t) = A_1 \exp(B_1 t) + A_2 \exp(B_2 t) \tag{8}$$

where t is time, A_1 and B_1 are the parameters of the first term of the approximation function, A_2 and B_2 are the parameters of the second term of the approximation function.

As a result of the curve fitting, a set of function parameters was estimated. The fitting results were estimated on the basis of the coefficient of multiple determination (R-squared) which did not fall below 96%. Data analysis shows that the first part of the self-heating curve is controlled by the second term of function $\theta(t)$ whereas the first term controls the second part. Pre-exponential parameters A_i are responsible for the temperature magnitudes in both terms of the curve whilst parameters B_i determine the heating rate. Presented in Figure 4 is the exemplary fitted curve of temperature evolution values.

It could be noticed that the obtained temperature curves consisted of two regions. In the first region, the heating-up of the structure is observed, while in the second one we observed a region of temperature stability. The analytical model presented above considers the heating of the structure and temperature stabilization. During the heating-up of the structure, the evolution of dynamic moduli occurs. As mentioned above, the amount of dissipated energy depends on the stress magnitudes (see (6)): when Q_{sh} is greater than the convected energy, temperature growth is observed and when the amounts of these energies are equal, the system reaches the thermal steady-state, which is illustrated by the solid line in Figure 4.

The evolution of the dynamic moduli (storage and loss modulus) could be estimated from the master curve which presents the variation of a given complex parameter dependent on the temperature and excitation frequency (the master curve for the loss modulus for the investigated composite was presented in [13]) or from the dynamic tests presented in [10] using (4). During the heating-up of the structure, the storage modulus decreases and the loss modulus increases until it reaches a thermal steady-state which cause a decrease in stiffness of the structure. Moreover, this phenomenon causes the shifting of FRF and the movement of the excitation frequency beyond the resonant frequency, thus the structure is excited with values of less amplitude. The resonant frequency magnitudes decrease due to material softening during heating of the structure. The phenomenon of FRF shift was also observed by the authors of [9]. However, in the present study, due to the small temperature growth in the experiments, the influence of FRF shifting was insignificant.

Model parameters obtained on the basis of experimental investigation allowed us to identify the relation between the dimension of the specimens and the self-heating temperature growth. It was observed that for longer plates, the stress amplitudes during vibration decreased and the temperature gradient lowered (Fig. 7). It is in agreement with an intuitive look on this matter because as much specimen volume as energy was needed for the dissipation to occur, thus a smaller amount of energy was converted to heat. In the second considered case, where the influence of the changed excitation frequency on temperature growth was investigated, the relation of the temperature variation is not so obvious (Fig. 6). Two factors have an influence on the temperature values: on one hand lower stress magnitudes were observed for higher resonant frequencies, which induced a slower temperature increase, and on the other hand the loss modulus (which is in direct dependence with the self-heating temperature [11]) increases following the time-temperature superposition principle. The obtained models are in agreement with the DMA tests, which were carried out on the same material [13]. The parameters of model (7) for the considered cases were presented in Table 1.

TABLE 1. **Model parameters**
TABELA 1. **Parametry modelu**

	a [°C]	*b* [1/s]	*c* [°C]	*d* [1/s]
f_1	28,7	9,77E-06	−2,1	−0,01967
f_2	28,6	1,61E-05	−1,999	−0,01885
f_3	28,55	1,30E-05	−1,985	−0,0185
l_1	28,72	9,77E-06	−6,7	−0,01967
l_2	27,27	6,47E-07	−5,281	−0,01996
l_3	25,9	1,83E-05	−3,887	−0,02

The parameters of the model presented in Table 1 describe the initial temperature (result of subtraction of *a* and *c*) and the shape of the temperature evolution against the time (parameters *b* and *d*).

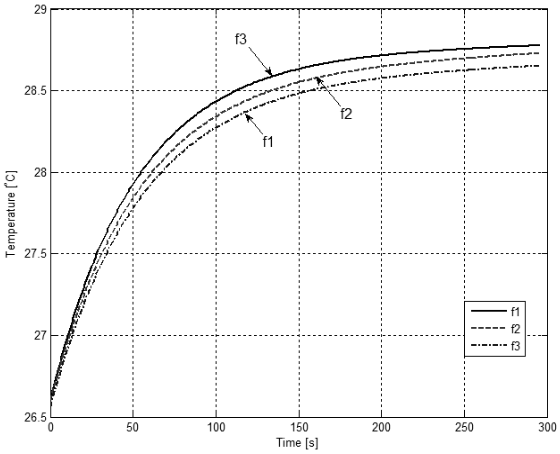

Fig. 6. Exemplary fitted temperature realizations for plates with L_e = = 180 mm evaluated from infrared images during excitation at resonant frequencies: f_1 = 262 Hz, f_2 = 739 Hz, f_3 = 1225 Hz

Rys. 6. Przykładowe zaproksymowane przebiegi temperatury dla płyty z L_e = 180 mm otrzymane z obrazów termowizyjnych podczas wymuszenia przy częstościach rezonansowych: f_1 = 262 Hz, f_2 = 739 Hz, f_3 = 1225 Hz

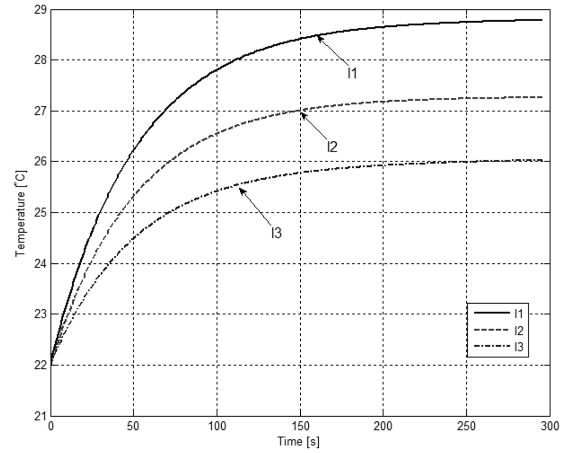

Fig. 7. Exemplary fitted temperature realizations evaluated from infrared images during excitation of plates with lengths L_e^1 = 180 mm, L_e^2 = 255 mm, L_e^3 = 280 mm at their first resonant frequency f_1 = 262 Hz, f_2 = 128 Hz, f_1 = 108 Hz

Rys.7. Przykładowe zaproksymowane przebiegi temperatury otrzymane z obrazów termowizyjnych podczas wymuszenia płyt o długościach L_e^1 = 180 mm, L_e^2 = 255 mm, L_e^3 = 280 mm przy pierwszych częstościach rezonansowych f_1 = 262 Hz, f_2 = 128 Hz, f_1 = 108 Hz

CONCLUSIONS

Temperature evolution during constant-strain oscillatory loading in a free air convection environment was observed. During the test, the temperature profiles and their evolution was evaluated and fitted using a double-exponential function. The temperature characteristics show that due to the evolution of dynamic moduli, the temperature increases to the equilibrium state between dissipated and convected energies and then decreases when the value of convected energy is higher than the dissipated one. Moreover, FRF shifting during the heating-up of the structure also influenced the self-heating effect due to material softening and changes of its complex parameters.

In the observed temperature range, the influence of eigen frequencies shifting was insignificant. In the presented study the authors also analyzed the influence of the excitation frequency and the specimen length on the temperature magnitude and its time evolution. The results show that self-heating temperature decreases with an increase in length. The influence of the excitation frequency was very small; however, the self-heating temperature variability could have resulted from lower stress rates and the evolution of the loss modulus. In order to verify the influence of excitation frequencies, additional tests will be performed where the specimens will be excited with higher stress amplitudes to obtain wider temperature ranges. The obtained results could be used to improve existing theoretical models and would be helpful in structural health monitoring applications for polymeric composite constructions. The results also could be helpful in the prediction and prevention of composite structural degradation.

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